**10 .... 0(2)=2^n**

**n**

**1... .... 1(2)= 10.....0(2) -1(2)= 2^n -1**

**n n**

**Integer numbers – codes and operations in complementary code**

**Example 1**

**n=8 bits**

**X= 105**

**Y= 53**

**X=105=64+32+8+1= 2^6+2^5+2^3+2^0=1101001(2)**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **positions** | **S 7 6 5 4 3 2 1 0** | | | | | | | |
| **[105] dir = [105]inv= [105]compl =** | **0** | **1** | **1** | **0** | **1** | **0** | **0** | **1** |
| **[-105]dir =** | **1** | **1** | **1** | **0** | **1** | **0** | **0** | **1** |
| **[-105]inv =** | **1** | **0** | **0** | **1** | **0** | **1** | **1** | **0** |
| **[-105]compl =** | **1** | **0** | **0** | **1** | **0** | **1** | **1** | **1** |

**Y=53=32+16+4+1=2^5+2^4+2^2+2^0=110101(2)**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **positions** | **S 6 5 4 3 2 1 0** | | | | | | | |
| **[53] dir = [53]inv= [53]compl=** | **0** | **0** | **1** | **1** | **0** | **1** | **0** | **1** |
| **[-53]dir =** | **1** | **0** | **1** | **1** | **0** | **1** | **0** | **1** |
| **[-53]inv =** | **1** | **1** | **0** | **0** | **1** | **0** | **1** | **0** |
| **[-53]compl =** | **1** | **1** | **0** | **0** | **1** | **0** | **1** | **1** |

**[105+53]compl = [105]compl  [53]compl**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S** | | | | | | | |  | **Overflow**  **The operands are positive, the result is negative** |
| **[105]compl =** |  | **0** | **1** | **1** | **0** | **1** | **0** | **0** | **1** | **** |
| **[53]compl =** |  | **0** | **0** | **1** | **1** | **0** | **1** | **0** | **1** |  |
|  |  | **1** | **0** | **0** | **1** | **1** | **1** | **1** | **0** |  |

**[105-53]compl = [105]compl  [-53]compl**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S** | | | | | | | |  | **Correct result**  **2^2+2^4+2^5=52** |
| **[105]compl =** |  | **0** | **1** | **1** | **0** | **1** | **0** | **0** | **1** | **** |
| **[-53]compl =** |  | **1** | **1** | **0** | **0** | **1** | **0** | **1** | **1** |  |
| **[52]compl** | **~~1~~** | **0** | **0** | **1** | **1** | **0** | **1** | **0** | **0** |  |

**[53-105]compl = [53]compl  [-105]compl**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S** | | | | | | | |  | **Correct result** |
| **[53]compl** |  | **0** | **0** | **1** | **1** | **0** | **1** | **0** | **1** | **** |
| **[-105]compl** |  | **1** | **0** | **0** | **1** | **0** | **1** | **1** | **1** |  |
| **[-52]compl** |  | **1** | **1** | **0** | **0** | **1** | **1** | **0** | **0** |  |
| **[52]compl** |  | **0** | **0** | **1** | **1** | **0** | **1** | **0** | **0** |  | **complement** |

**Subunitary numbers – codes and operations in complementary code**

**Example 2**

**n=8 bits**

**X= 0,55 = 0,1000110(2)**

**Y= 0,63**

**X= 0,55**

**0,55 \* 2 = 1,1**

**0,1 \* 2 = 0,2**

**0,2 \* 2 = 0,4**

**0,4 \*2= 0,8**

**0,8 \*2 = 1,6**

**0,6 \* 2 = 1,2**

**0,2 \* 2 = 0,4**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **positions** | **S 7, 6 5 4 3 2 1 0** | | | | | | | |
| **[0,55] dir = [0,55]inv= [0,55]compl =** | **0** | **1** | **0** | **0** | **0** | **1** | **1** | **0** |
| **[-0,55]dir =** | **1** | **1** | **0** | **0** | **0** | **1** | **1** | **0** |
| **[-0,55]inv =** | **1** | **0** | **1** | **1** | **1** | **0** | **0** | **1** |
| **[-0,55]compl =** | **1** | **0** | **1** | **1** | **1** | **0** | **1** | **0** |

**Y= 0,63=0,1010000(2)**

**0,63\*2 = 1,26**

**0,26\*2 = 0,52**

**0,52\*2 = 1,04**

**0,04 \*2=0,08**

**0,08\*2 = 0,16**

**0,16\*2=0,32**

**0,32\*2=0,64**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **positions** | **S 7, 6 5 4 3 2 1 0** | | | | | | | |
| **[0,63] dir = [0,63]inv= [0,63]compl=** | **0** | **1** | **0** | **1** | **0** | **0** | **0** | **0** |
| **[-0,63]dir =** | **1** | **1** | **0** | **1** | **0** | **0** | **0** | **0** |
| **[-0,63]inv =** | **1** | **0** | **1** | **0** | **1** | **1** | **1** | **1** |
| **[-0,63]compl =** | **1** | **0** | **1** | **1** | **0** | **0** | **0** | **0** |

**[0,55+0,63]compl = [0,55]compl  [0,63]compl**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S ,1 0 0 0 0 0 0 0** | | | | | | | |  |  |
| **[0,55]compl =** |  | **0** | **1** | **0** | **0** | **0** | **1** | **1** | **0** | **** | **Overflow**  **Operands are positive, result is negative** |
| **[0,63]compl =** |  | **0** | **1** | **0** | **1** | **0** | **0** | **0** | **0** |  |
|  |  | **1** | **0** | **0** | **1** | **0** | **1** | **1** | **0** |  |

**[0,63-0,55]compl = [0,63]compl  [-0,55]compl**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S ,1 1 1 0 0 0 0 0** | | | | | | | |  |  |
| **[0,63]compl =** |  | **0** | **1** | **0** | **1** | **0** | **0** | **0** | **0** | **** | **Correct result**  **0\*2^ (-1) +0\*2^ (-2) +0\*2^ (-3) +1\*2^ (-4) +0\*2^ (-5) +1\*2^ (-6) +0\*2^ (-7) = 1\*2^ (-4) 1\*2^ (-6) = 1/16+1/64=5/64=0,078 aprox 0,08** |
| **[-0,55]compl =** |  | **1** | **0** | **1** | **1** | **1** | **0** | **1** | **0** |  |
| **[0,078]compl=** | **~~1~~** | **0** | **0** | **0** | **0** | **1** | **0** | **1** | **0** |  |

**[0,55-0,63]compl = [0,55]compl  [-0,63]compl**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **S,0 0 0 0 0 0 0 0** | | | | | | | |  |  |
| **[0,55]compl =** |  | **0** | **1** | **0** | **0** | **0** | **1** | **1** | **0** | **** | **Correct result**  **-(2^(-4)+2^(-6))=**  **=-0,078** |
| **[-0,63]compl =** |  | **1** | **0** | **1** | **1** | **0** | **0** | **0** | **0** |  |
| **[-0,078]compl=** |  | **1** | **1** | **1** | **1** | **0** | **1** | **1** | **0** |  |
| **[0,078]compl=** |  | **0** | **0** | **0** | **0** | **1** | **0** | **1** | **0** |  | **complement** |

**Example 3: Represent in fixed-point notation, on 32 bits, I=14 bits, the number 3456,78**

**1+I+F=32bits, F=17bits**

**3456,78= 110 110 000 000 , 101 001 110 010 110 011 (2)**

**3456=6600(8)=110 110 000 000(2)**

**3456/8=432, r=0**

**432/8=54, r=0**

**54/8=6, r=6**

**6/8=0, r=6**

**0,78=0,617273(8)= 0,101 001 110 010 110 011(2)**

**0,78 \* 8 = 6,24**

**0,24 \* 8 = 1,92**

**0,92 \* 8 = 7,36**

**0,36 \* 8 = 2,88**

**0,88 \* 8 = 7,04**

**0,40 \* 8 = 3,20**

**3456,78= 110 110 000 000 , 101 001 110 010 110 011 (2)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | |  | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | | I=14 bits -> , <- F=17bits | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 0 | | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 1 | | | | | B | | | | 0 | | | | 1 | | | | 4 | | | | E | | | | 5 | | | | 9 | | | |

**Example 4: Represent in floating point notation, single precision (SP), the number: 3456,78**

**Mantissa<1**

**3456,78= 110 110 000 000 ,101 001 110 010 110 011 (2)=**

**= 0,110 110 000 000 101 001 110 010 110 011 (2)\*2^12**

**mantissa**

**e=12, c=12+127=139=128+8+2+1=1 0 0 0 1 0 1 1(2)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | -> , <- | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | | C(8 bits) , c=e+127 | mantissa(23 bits) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 1 | | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 |
| 4 | | | | | 5 | | | | E | | | | C | | | | 0 | | | | 5 | | | | 3 | | | | 9 | | | |

**Mantissa>1**

**3456,78= 110 110 000 000 , 101 001 110 010 110 011 (2)=**

**= = 1,10 110 000 000 101 001 110 010 110 011 (2)\*2^11**

**mantissa**

**1 – hidden bit**

**e=11, c=11+127=138=128+8+2=1 0 0 0 1 0 1 0(2)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | -> , <- | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | | C=e+127 (8 bits) | mantissa(23 bits) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 1 | | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 4 | | | | | 5 | | | | 5 | | | | 8 | | | | 0 | | | | A | | | | 7 | | | | 2 | | | |

**Example 5: Find the real number X having C504A800 its fixed-point representation on 32 bits with I=17 bits.**

**C504A800 (16)= 1100 0101 0000 0100 1010 1000 0000 0000 (16)**

**X=- 10000101000001001 0, 10 1000 0000 0000(2)=**

**= -(2^1 + 2^4 + 2^10 + 2^12 + 2^17+ 2^-1 2^-3 \*1 =**

**= -70674,625**

**100001010000010010 (2)= -70674**

**0,10 1000 0000 0000 = 2^-1 \* 1 + 2^-2 \* 0 + 2^-3 \* 0 = ½ + 1/8 = 0.625**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | I = 17 bits ->, <- F = 14 bits | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | | | | 5 | | | | 0 | | | | 4 | | | | A | | | | 8 | | | | 0 | | | | 0 | | | |

**Example 6:** **Find the real number X having C504A800 as its floating-point representation, SP, m>1.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | -> , <- | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | | c=e+127 (8 bits) | mantissa(23 bits) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | | | | | 5 | | | | 0 | | | | 4 | | | | A | | | | 8 | | | | 0 | | | | 0 | | | |

c = 10001010(2) = 2^1 + 2^3 + 2^7 = 2+ 8 + 128 = 138 ,

c=e+127=138 , e = 11

1- hidden bit

X = -1, 00001001010100000000000 \*2^11 =-100001001010,1(2) =

= 2 + 2 ^ 3+ 2 ^ 6 + 2^11 + 2^-1 = 2 + 8 + 64 + 2048 + 0,5

= 74 + 2048 ,5 = 2122,5